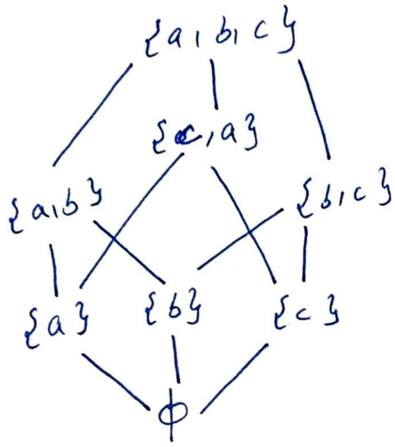
5. HASSE DIAGRAM:

A Graphical representation of a Partial order relation in which all arrowhead are understood to be Pointing upward is Known as Hasse Diagram.

e.g 1. The Hasse Diagram of subsets of $S = \{a, b, c\}$ with inclusion relation '$\subseteq$'.

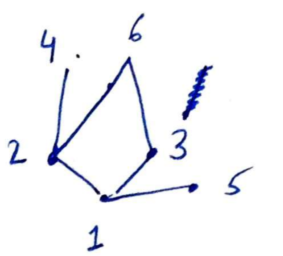
Sol:- subsets of $S = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$



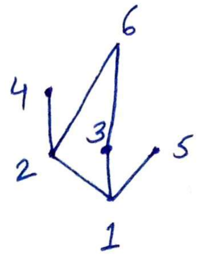
e.g 2. Draw the Hasse Diagram for Poset

i) $\{\{1, 2, 3, 4, 5, 6\}, /\}$

Sol:-



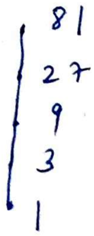
or



q.4 Let A = {1, 2, 3, 4}   R = {(1,1) (2,1) (2,2) (2,1)   9. 8.

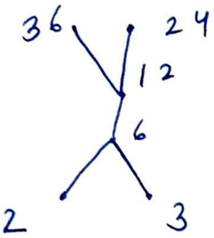
ii) {{1, 3, 9, 27, 81}, /}

Sol:-



iii) {{2, 3, 6, 12, 24, 36}, /}

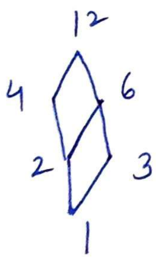
Sol:-



q.3 Draw Hasse Diagram of

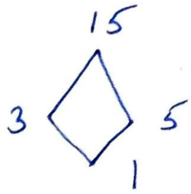
i) D_{12}

Sol:- D_{12} = {1, 2, 3, 4, 6, 12}



ii) D_{15} = {1, 3, 5, 15}

Sol:-



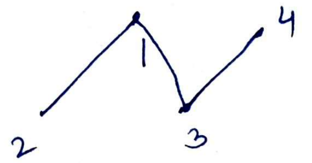
y.10 X = { 2, 3, 5 }

y.4 Let A = { 1, 2, 3, 4 } R = { (1,1) (2,1) (2,2) (3,1), (3,3) (3,4) (4,4) }

R is Partial ordering Draw Hass diagram.

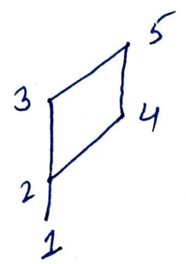
[Ist No. ⇒ Down
2nd No. ⇒ up]

sol.



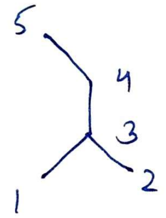
y 5 A = { 1, 2, 3, 4, 5 } Determine the relations by Hasse Diagram:

i)



Sol. R = { (1,2) (1,3) (1,4), (1,5), (1,1), (2,3), (2,5), (2,4), (2,2), (3,3), (3,5), (4,4) (4,5), (5,5) }

ii)



Sol :- R = { (1,1), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,4), (3,5), (4,4), (4,5), (5,5) }

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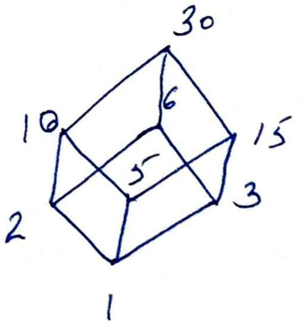
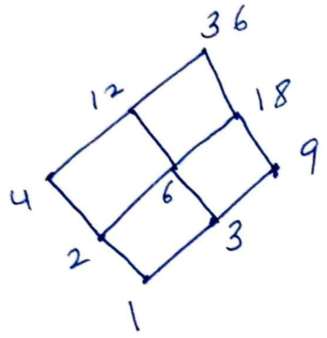
eg. 6 $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

Sol :-



eg 7 $D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

Sol :

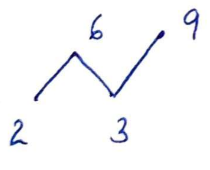


☆ Trivial Divisor :- A number is either 1 or the number itself.

e.g. Trivial divisor of 15 = $\pm 1, \pm 15$.

eg. 8 {Non- Trivial Divisor of 18} Draw its Hasse Diagram.

$= \{2, 3, 6, 9\}$

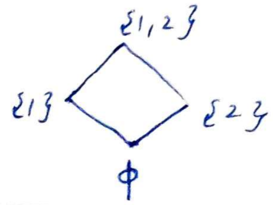


eg. 9 $X = \{1, 2\}$

$P(X)$ = Power Set = $\{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Draw its Hasse Diagram.

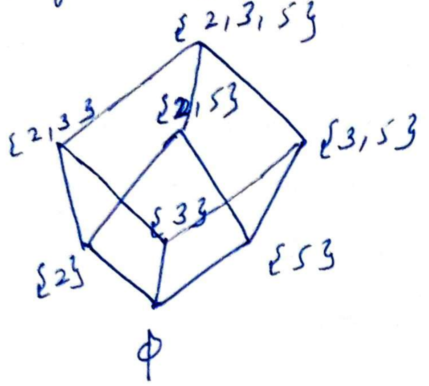
Sol :-



Q.10 $X = \{2, 3, 5\}$

$P(X) = \{\phi, \{2\}, \{3\}, \{5\}, \{2,3\}, \{2,5\}, \{3,5\}, \{2,3,5\}\}$

Sol Hasse Diagram of $P(X)$:-



6. Maximal, Minimal, Greatest And Least Elements :-

i) Maximal Elements :- • Top Elements of the Hasse diagram.

• elements having no successor.

• An element $a \in A$ is called maximal if there is no element in A s.t. $a \leq b$ (i.e. $(a, b) \notin R$)

ii) Minimal Elements :- • Bottom Elements of the Hasse diagram.

• Element having no Predecessor.

• An Element $a \in A$ is called minimal if there is no element in A s.t. $b \leq a$ ($(b, a) \notin R$)

iii) Greatest Element :- Let $(P, \leq)$ be Poset. If $\exists$ an element $a \in P$ s.t.
(maximum) (unique)

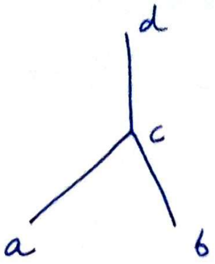
$x \leq a \quad \forall x \in P$

'a' is greatest Element of P.

• Greater than all Elements.

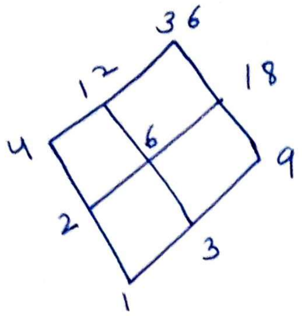
iv) Least Element :- An Element $a \in P$ is called least if $x \geq a$ or $a \leq x \quad \forall x \in P$. (unique)
(Minimum)

examples :- 1.



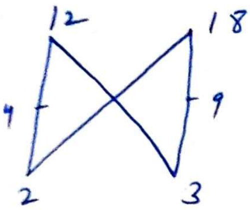
Maximal - d
Minimal - a, b
Greatest - d
Least - None

2.



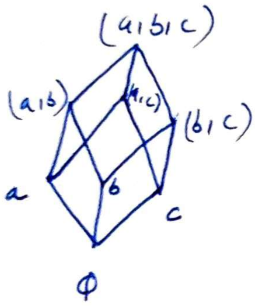
Maximal - 36
Minimal - 1
Greatest - 36
Least - 1

3.



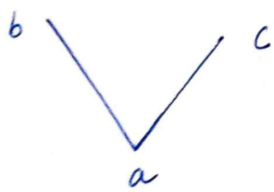
Maximal - 12, 18
Minimal - 2, 3
Greatest - None
Least - None

4.



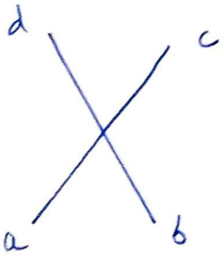
Maximal - (a, b, c)
Minimal - ϕ
Greatest - (a, b, c)
Least - ϕ

5.



Maximal - b, c
Minimal - a
Greatest - None
Least - a

6.



Maximal - c, d
Minimal - a, b
Greatest - None
Least - None